How speculation can explain the equity premium

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November 16, 2016

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- Returns from stocks better than returns from bonds.
- Attributed to risk aversion: stocks riskier, investors pay less.
- Game-theoretic explanation attributes premium to speculation.
- This better accounts for its size.

How speculation can explain the equity premium,
by Glenn Shafer

When measured over decades in countries that have been relatively stable, returns from stocks have been substantially better than returns from bonds. This is often attributed to investors' risk aversion. The game-theoretic probability-free theory of finance attributes the equity premium to speculation, and this explanation does better than the explanation from risk aversion in accounting for the magnitude of the premium.

This is Working Paper 47 at www.probabilityandfinance.com. Direct link is http://www.probabilityandfinance.com/articles/47.pdf.

The equity premium puzzle

In 2008 review, Mehra & Prescott reported that returns from stocks were more than 6 percentage points better than returns from bonds in US from 1889 to 2005.

As they first showed in late 1970s, explanations based on risk aversion cannot account for this difference.

Standard theory justifies only about 1 percentage point.

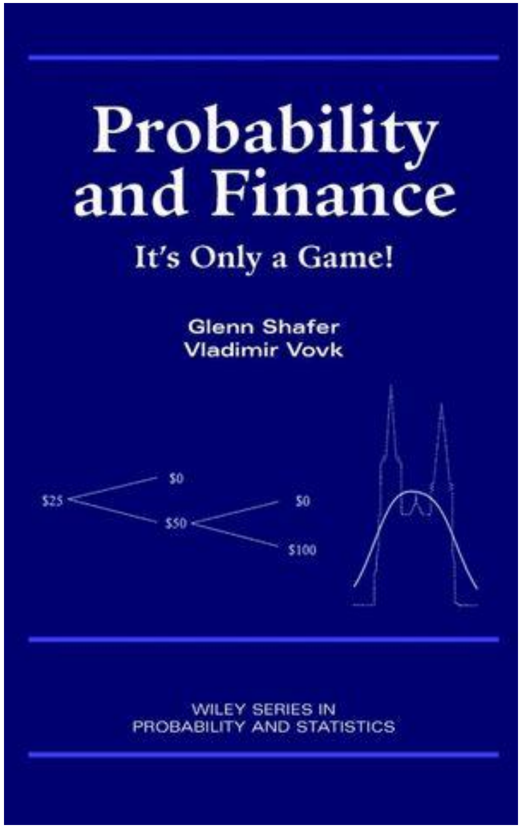
Game-theoretic explanation of equity premium

- Speculation causes volatility.
- Speculation makes market efficient.
- Speculation forces an efficient market to appreciate in proportion to the square of its volatility.

2001 book

Game-theoretic probability and finance.

- Start with a game, not with a probability model.
- Probabilities emerge from the game.



15 years of subsequent working papers at www.probabilityandfinance.com.

Second edition (or rather new book) in preparation.

Three roles of speculation

- Speculation causes volatility.
  Traders know this, though the academic literature wants to attribute volatility to information.

- Speculation makes market efficient.
  Conventional wisdom, even in academia.

- Speculation forces an efficient market to appreciate in proportion to $(\text{volatility})^2$.
  This is our theoretical contribution.

Three roles of speculation

- Speculation causes volatility. Traders and experts in option pricing agree.

- Speculation makes the market efficient by exhausting opportunities for low-risk profit. An investor can rarely do better than hold all tradables in proportion to their capitalization.

- Assuming that you can trade an index that holds all tradables in proportion to their capitalization, speculation forces this index to appreciate in proportion to the square of its volatility.

John Hull, author of leading textbook on option pricing:

What Causes Volatility?

It is natural to assume that the volatility of a stock is caused by new information reaching the market. This new information causes people to revise their opinions about the value of the stock. The price of the stock changes and volatility results. This view of what causes volatility is not supported by research.

The only reasonable conclusion is that volatility is to a large extent caused by trading itself. (Traders usually have no difficulty accepting this conclusion.)

What is an efficient market?

- Fama 1965: Prices incorporate all information.
- Shafer/Vovk 2001: No strategy selected in advance multiplies capital risked by large factor.

Why should a market be efficient?

- Fama: Speculators use each bit of new information.
- Shafer/Vovk: Speculators are using every trick to multiply their capital, not merely exogenous information.

How do we test whether a market is efficient?

- Fama: Postulate a model and test it statistically.
- Shafer/Vovk: Try to multiply your capital in the market.

How do we test whether a market is efficient?

Try to multiply your capital in the market.

- Define a trading strategy and implement it.
- If you multiply your money by 1000, reject the hypothesis of efficiency.
- Confidence of rejection same as when you reject a hypothesis at significance 0.001.

THE EFFICIENT INDEX HYPOTHESIS (EIH)

You will not multiply the capital you risk by a large factor relative to an index defined by the total value of all the readily tradable assets.

To fix ideas, suppose the index is the S&P500.

ETF Symbol	ETF Name	Fees, per year
IVV	iShares Core S&P 500	4 bps
SPY	SPDR S&P 500	11 bps
VOO	Vanguard S&P 500	5 bps

Our mathematical story

We have argued that speculation causes volatility, and that speculation makes the market efficient, in the sense that <u>the market index will not be beat</u>.

This is <u>the efficient index hypothesis</u>.

Using the efficient market hypothesis, we now prove mathematically that <u>the market index must grow in proportion to the variance of the index</u>.

Assume zero interest rate.

For traders, “cash” is a money-market account that pays the short-term risk-free interest rate.

Use the accumulated value of $1 in such an account as the ***numéraire*** for measuring the value of other financial instruments.

Mathematically, this is equivalent to assuming that the interest rate is zero.

Efficient Index Hypothesis (EIH)

You will not multiply the capital you risk by a large factor relative to an index I.

Volatility and Variance

Suppose the value of the index over N times periods is $I_0, I_1, \ldots, I_N$. The *returns* $m_1, m_2, \ldots, m_N$ are defined by

$$m_n := \frac{I_n}{I_{n-1}} - 1 = \frac{I_n - I_{n-1}}{I_{n-1}}.$$

for $n = 1, \ldots, N$. The *relative quadratic variation* is

$$\Sigma_N := \sum_{n=1}^{N} m_n^2.$$

The *cumulative volatility* is $\sqrt{\Sigma_N}$.

Measure time by accumulated variance.

To fix ideas, consider daily returns, so N is the number of days.

The relative quadratic variation Σ_N is usually approximately proportional to the amount of time elapsed.

- Explanation by probability theory: The m_n are random and independent. Each is random with mean 0 and standard deviation σ. So $\Sigma_N := \sum_{n=1}^{N} m_n^2$ is an estimate of $N\sigma^2$.
- Same conclusion from EIH without probability assumptions.

But volatility does vary (σ changes). It is greater when traders get excited, for whatever reason.

To keep the mathematics simple, we use Σ_N as our clock!! In other words, we measure time by the amount of trading.

Pass to continuous time

Makes picture mathematically elegant.

- Mathematical finance now uses measure-theoretic continuous-time probability.
- Instead, we use game-theoretic continuous-time probability.

Assuming continuous time...

- Measure time by cumulative variance $\Sigma = \sum m_n^2$.
- Write I_s for the value of index at time s, where $s = \Sigma$.
- Assume $I_0 = 1$.

Then the EIH implies that I_s will look like

$$I_s = \exp\left(\frac{s}{2} + W_s\right),$$

where W_s is Brownian motion.

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where W_s is Brownian motion.

Geometric Brownian motion with drift 1 & volatility 1.

$$\mathbf{E}(\ln I_s) = \frac{s}{2}$$

$$\mathbf{s.\,d.}(\ln I_s) = \sqrt{s}$$

For large s,

$$\ln I_s \approx \frac{s}{2}.$$

When time is measured by cumulative variance Σ,

$$\ln I_s \approx \frac{s}{2}, \text{ where } s = \Sigma.$$

or

$$\ln I_\Sigma \approx \frac{\Sigma}{2}.$$

In terms of calendar time t.

$$\ln I_t \approx \frac{\Sigma_t}{2},$$

where Σ_t is the cumulated variance at time t.

Average return overestimates growth in value by half the variance.

Suppose m_n is the return for period n,

$$\mathrm{M} = \sum_n m_n, \qquad \Sigma = \sum_n m_n^2,$$

and I is the accumulated value when you start with one unit. Then

$$\ln I = \ln \prod_n (1 + m_n) = \sum_n \ln(1 + m_n).$$

By the Taylor expansion $\ln(1 + x) \approx x - \frac{1}{2}x^2$,

$$\ln I \approx \sum_n \left(m_n - \frac{1}{2} m_n^2 \right) = M - \frac{\Sigma}{2}.$$

By our theory (still to be explained),

$$\ln I_t \approx \frac{\Sigma_t}{2},$$

By the properties of the logarithm,

$$M_t \approx \ln I_t + \frac{\Sigma_t}{2}.$$

So

$$M_t \approx \Sigma_t$$

This is the equity premium.

$$M_t \approx \Sigma_t .$$

The annualized volatility of the S&P 500 is approximately 20% ([10], page 8). Squaring this, we obtain an equity premium of 4%. This is closer to the empirical estimates than the 1% obtained from standard theory, and GTP38 shows that it is within (3)'s anticipated error of approximation (Section 4).

$$\mathrm{M}_t \approx \Sigma_t.$$

How does the EIH implies this equity premium?

Answer: There are strategies that can beat the index (multiply your capital by a large factor relative to the index) if the approximation does not hold.

The trading strategy

Suppose I grows faster than our theory predicts:

$$\mathrm{M}_t >> \Sigma_t.$$

To make money, you invest all you have in I and borrow money to invest even more.

Say you always invest $(1+\epsilon) \times$ (current capital) in I. Then on round n, when I is multiplied by $1+m_n$, your capital is multiplied by $1+(1+\epsilon)m_n$. Relative to I, your capital is multiplied by

$$\frac{1+(1+\epsilon)m_n}{1+m_n}.$$

Use Taylor's series for the logarithm again:

$$\begin{aligned} \ln \frac{1+(1+\epsilon)m_n}{1+m_n} &= \ln(1+(1+\epsilon)m_n) - \ln(1+m_n) \\ &\approx \epsilon m_n - \epsilon m_n^2 - \frac{\epsilon^2}{2} m_n^2. \end{aligned}$$

multiplied by $1+(1+\epsilon)m_k$. So your capital will increase (or decrease) relative to I by the factor

$$\frac{1+(1+\epsilon)m_k}{1+m_k}.$$

Using Taylor's series for the logarithm, we obtain the approximation

$$\ln\frac{1+(1+\epsilon)m_k}{1+m_k} \approx \epsilon m_k - \epsilon m_k^2 - \frac{\epsilon^2}{2}m_k^2.$$

So over K rounds, your capital will grow relative to I by a factor whose logarithm is approximately

$$\begin{aligned}\epsilon\sum_{k=1}^{K} m_k - \epsilon\sum_{k=1}^{K} m_k^2 - \frac{\epsilon^2}{2}\sum_{k=1}^{K} m_k^2 &= \epsilon \mathrm{M}_t - \epsilon\Sigma_t - \frac{\epsilon^2}{2}\Sigma_t \\ &= \epsilon(\mathrm{M}_t - \Sigma_t) - \frac{\epsilon^2}{2}\Sigma_t, \qquad (5)\end{aligned}$$

So over the entire time period, your capital will grow relative to I by a factor whose logarithm is approximately

$$\epsilon \sum_n m_n - \epsilon \sum_n m_n^2 - \frac{\epsilon^2}{2} \sum_n m_n^2 = \epsilon \mathrm{M}_t - \epsilon \Sigma_t - \frac{\epsilon^2}{2} \Sigma_t$$
$$= \epsilon(\mathrm{M}_t - \Sigma_t) - \frac{\epsilon^2}{2} \Sigma_t.$$

This factor will be large if you continue until Σ_t is so large that $\epsilon\Sigma_t$ is large even though ϵ is small, and if M_t then exceeds Σ_t substantially.

Example: $\epsilon = 0.01$, $\epsilon\Sigma_t = 3$, $\mathrm{M}_t \approx 1.5\Sigma_t$. Then

$$\epsilon(\mathrm{M}_t - \Sigma_t) - \frac{\epsilon^2}{2} \Sigma_t \approx 1.5,$$

so that you have multiplied your capital relative to I by $e^{1.5} \approx 4.5$.

To similarly make money if I grows too slowly—i.e., if M_t is substantially less than Σ_t, you can take ϵ in the preceding argument to be a small negative number. In other words, you keep a small fixed fraction of your capital in the risk-free bond on each round, investing the rest in I.

You can implement the two strategies simultaneously: put half your initial capital on one of them and half on the other. So you have a strategy that will mulitply its initial capital substantially relative to I unless $\mathrm{M}_t \approx \Sigma_t$. (We promised a strategy that multiplies *the capital it risks*, so you need to implement the strategy just sketched in a way that risks no more than its initial capital. You can do this by stopping the strategy if its capital gets close to zero. In GTP44 we rely on the assumption that the price path is continuous to make sure we can stop in time. Weaker assumptions can also be accommodated.)

What are the macroeconomic implications?

The market represents one portion of society's productive capital—the portion that is so liquid that we can speak of its volatility. How do changes in the valuation of this portion of society's capital relative to cash[7] affect its valuation relative to the portion of society's productive capital that is not so liquid?

Consider an extended period in which the publicly traded portion of the economy is exceptionally productive, so that the value of I is growing because of economic fundamentals at a rate exceeding its volatility. A naive expectation is that this exceptional growth will draw investors into the market, creating a demand for cash. The speculative strategy described in Section 4.3, if widely played, would reinforce this demand for cash. Half of its initial capital is invested according to the strategy (4), and since its current capital will grow, the amount $\epsilon \times$ (current capital) that it borrows will grow. The other half of its initial capital will be invested according to

$$(1 - \epsilon) \times (\text{current capital}),$$

but this current capital will grow more slowly, and so the amount $\epsilon \times$ (current capital) that it keeps in cash will be growing more slowly. The demand for cash will presumably drive up the interest rate. Since the higher interest rate will not be justifed by the productivity of capital outside the market, this might spur inflation. Restoration of stability might require slowing the productivity of the publicly-traded portion of the economy relative to the privately held portion, perhaps by taking some corporations private.

In an extended period of slow productivity for the publicly traded portion of the economy, which falls short of justifying an increase in capitalization commensurate with volatility, we would see pressures in the opposite direction. As the growth in I lags its volatility, the strategy described in Section 4.3, perhaps together with more aggressive strategies that sell all or parts of the market short, could produce an excess supply of cash, driving down the interest rate and creating deflationary pressure. As this process continues, wealth would be increasingly concentrated in the hands of those who own the assets in the market. Barring an increase in productivity, these pressures might eventually be released by a financial crisis that violates the EIH, suspending consequences such as $\mathrm{M} \approx \Sigma$ and perhaps durably destroying market capitalization that is not producing a commensurate flow of goods and services.