

This is an unusual and exciting paper, which promises to resolve many longstanding conundrums. I applaud the ingenuity and clear-sightedness of the author, and I look forward to his further contributions.

Vovk justly emphasizes his debt to A.P. Dawid. I must thank Vovk for making Dawid's work more accessible. I had not been able to appreciate Dawid's ideas fully until I saw Vovk draw the picture. The key seems to be the finitary character of Vovk's version of the principle of the excluded gambling strategy. Dawid's story had seemed hopelessly stuck at infinity, but Vovk has placed it in the finite world of applications.

I myself have advanced a finitary version of the principle of the excluded gambling strategy as a foundation for both the objective and subjective aspect of probability (Shafer 1992). Vovk's work deepens my story so beautifully that I must contest his assertion that his work concerns only the objective aspect. The subjective aspect is not a matter of psychology, or who believes in what. It is a matter of limits on a person's knowledge. The person does not know how to choose a non-negative  $\pi$ -supermartingale that will make money for her.

Sometimes it is this claim, rather than a claim about frequency, that is rejected when  $\pi$  is falsified. I see it as one virtue of the finitary approach that it does make the subjective aspect of probability salient. There is not enough time to wash away possible differences in knowledge by cycling through all lower semicomputable non-negative  $\pi$ -supermartingales.

As an illustration of the virtues of recognizing the subjective aspects of Vovk's story, let me point to how this dispels the misapprehension that the tree-like structure of  $\Omega^*$  is a restriction. It is not a restriction because  $\Omega^*$  is a picture of ways our knowledge might unfold rather than a picture of a stochastic object in itself (cf. Loveland 1966). Even if we are concerned with an inherently multi-dimensional object, as in geostatistics, our knowledge must unfold in time.

I conclude with a query. One of the beautiful aspects of Vovk's story is how naturally upper probabilities appear, as a consequence of the inevitable placement

of probabilistic knowledge in a larger unprobabilized world. To what extent does his apparatus generalize to the case where the  $\pi(x)$  themselves are only upper envelopes?

## References

Loveland, D. (1966) A new interpretation of the von Mises' concept of random sequence. *Zeitschr. f. math. Logik und Grundlagen d. Math.* **12** 279-294.

Shafer, G. (1992) Can the various meanings of probability be reconciled? In *A Handbook for Data Analysis in the Behavioral Sciences: Methodological Issues* (edited by Gideon Keren and Charles Lewis), pp. 165-196. Hillsdale, New Jersey: Lawrence Erlbaum.