A Logic of Action, Causality, and the Temporal Relations of Events

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Abstract

In this paper, we propose a logic of action and causality. The most important part of our contribution is a semantics that integrates action, temporal structure, and probability. The semantics is based on a mathematical structure called an event space. This space is inhabited by situations or instantaneous events with binary relations over them. The temporal relations within an event space include several relations that different authors have labeled "cause." Our language does not single out any one of them under this name and therefore avoids any appearance of reliance on some mysterious relation of causality that goes beyond the facts about the world and about how its evolution is best predicted.

1 Introduction

It is now widely recognized that planning, prediction, diagnosis, and many other tasks in artificial intelligence require a language in which actions and other causes can be represented. Ideally, this language should be a genuine logic, with a workable inference mechanism and a clear semantics. It could then be called a causal logic. In this paper, we join in the current effort to construct causal logics. We seek to represent the variety of relations between events that different authors have labeled "cause." The most important part of our contribution is a semantics that integrates action, temporal structure, and probability.

Our new semantics—our temporal ontology, as we will call it—is based on the intuition of branching continuous time, but the states that evolve in time, the *situations*, are not fully detailed states of the world. Indeed, the fundamental novelty of the ontology is its modest concept of a situation. Previous temporal ontologies have been formulated in terms of states that fully specify everything about the world, including a metric time. In our ontology, a situation never tells everything about the world. It can be refined so as to tell more, but we consider more refined situations only as the need arises. Probabilities for how a given aspect of a situation will evolve typically depend on certain other aspects of the situation but not on all details that might be added to a refinement of the situation. An action is simply a transition from one situation to a second situation, and hence we can speak also of refining an action; one action is a refinement of another if it has the same initial situation and a more refined final situation.

Mathematically, our temporal ontology begins with a set of objects called situations. The set is called an *event space*. The situations in it are related by certain binary relations and certain constructions, which satisfy certain axioms. The binary relations include refinement (situation T is more refined than situation S), precedence (situation T is possible in situation S and can only occur after S), and necessity (in situation S, situation T is inevitable). The axioms are derived from a rigorous analysis of the simple and well-understood idea of an event tree, and hence they constitute a clear and rigorous mathematization of the idea of branching time [14].

The event-space ontology has a number of advantages over more familiar mathematical structures for branching time:

1. Assumptions of persistence can be expressed both probabilistically and in terms of preconditions and excluded actions. Thus both probabilistic decision-theory methods and nonmonotonic reasoning methods can be made available within a single language.

2. Since a situation is not necessarily identified with a point on a metric time scale, the problem of the divided instant [16] does not arise.

3. The situations that form the branching time structure can also be thought of as instantaneous

events: the situation is the same as the instantaneous event that brings it into existence. Thus a language based on the ontology can also function as a logic of events.

4. Since situations and hence actions can have refinements within the ontology, we can represent both a nondeterministic action (tossing a coin, for example) and a more refined deterministic action (tossing the coin and getting heads, for example).

5. The binary relations in the ontology, which can be reproduced in a language based on it, include relations of necessity and possibility between situations or instantaneous events. A statement that one event is possible or inevitable (or "caused," as some authors want to say) after another thus has a direct meaning in the ontology.

6. The temporal relations within an event space include several relations that different authors have labeled "cause" [15, 7, 8]. By using all these relations and avoiding labelling any one of them "cause", we avoid the sense of mystery and metaphysics associated with this word. Our causal relations are specific and down-to-earth, saying something the actual world and how its evolution is best predicted. To use the words of David Lewis [6], these causal relations are supervenient on the bare facts about what happens.

Since our ontology is essentially a mathematical structure for branching time, we can construct a language that uses it in much the same way that established temporal logics use simpler discrete branching-time structures. Such a language can include names for situations, for actions, for individuals who may or may not exist in a certain situation, and for properties of these individuals that may or may not be specified in a certain situation. (In some cases, it may be necessary to refine a given situation in order to get values for a certain property.) Shafer [13] proposes a language similar to CLT* ([1]) except that instead of two kinds of proposition formulae (for propositions that are true or false in situations and propositions that are true or false of paths, respectively), one has proposition formulae (to represent propositions that are true, false, or indefinite in situations) and event formulae (to represent situations themselves, thought of as instantaneous events). Due to constraints of space, we omit event formulae in the language sketched here, but we do consider formulae that represent actions.

We believe that our ontology and languages built on it provide an advance that can be used within all the currents efforts to found causal reasoning on a branching-time picture, including efforts that already use a mathematical representation of branching time explicitly in their semantics (this is the tradition of temporal logic; see [17]) and those that use a sorted first-order logic and build the branching-time structure syntactically, by specifying permitted sequences of actions (this is the tradition of the situation calculus [9, 2, 10, 5]).

2 Event Trees and Event Spaces

Event and probability trees have been used since the seventeenth century. An event tree is simply a probability tree without the probabilities. Recently [13, 14, 12] these ideas have been further refined mathematically and developed into a foundation for probability and causality. Event trees show the different ways events may unfold.

As an illustration of a probability tree, consider Figure 1. which is a probability tree for whether Dennis, a twelve-year old boy, will remember to practice his saxophone before dinner on a summer afternoon. He is least likely to do so when his friend Alex comes to his house and the two boys then go to Alex's house. He is most likely to do so when he stays at home by himself and reads. The probabilities and contingencies in the tree are in reference to an idealized observer who watches the events as they unfold. At each point the observer's limited ability to predict what will happen next is indicated by the probabilities attached to the branches at that point. Here it is natural to say that watching television contributed to (or caused) Dennis's not practicing. Notice that the nodes (circles) in the tree represent situations. Individuals (such as Dennis) have different properties (such as location) in different situations. The situations can also be thought of as instantaneous events; the node labelled S in Figure 1 can be thought of as the situation where Dennis has just arrived at Sigmund's house or as the event of his arrival.

Event and probability trees often need to be refined. Figure 2 illustrates this point. There the simpler tree on the right agrees with the probabilities given by the more refined tree on the left; both give the same initial probability for Rick's eventually reading ($.8 \times .8 \times .625 = .4$) and for his eventually watching television ($.2 + .8 \times .2 + .8 \times .8 \times .375 = .6$). Every situation in the simpler tree is also represented in the more refined tree. The situation represented by the node labelled R on the right is also represented by a node labelled R on the left. The situation represented by the node labelled S on the right is represented on the left by a collection of three nodes, S_1, S_2, S_3. For a

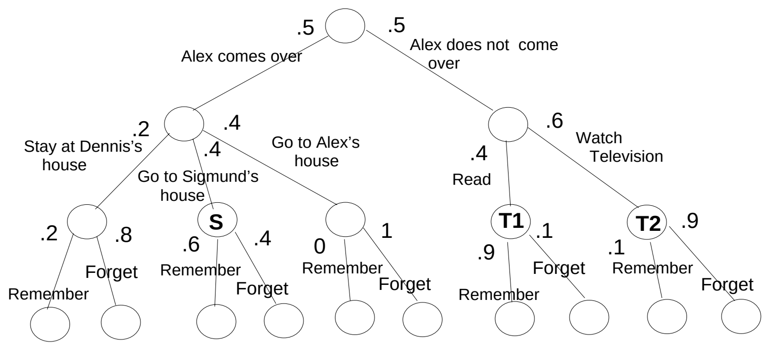

Figure 1: Will Dennis remember to practice his saxophone?

more detailed and rigorous account of refinement, see Chapter 13 of [12].

In general, as we have just seen, a situation can be represented in an event tree either by a single node or by a *clade* of nodes—a set of nodes that are divergent, in the sense that no two are on the same path down the tree. The same situation represented by a node in one tree is represented by a clade in another. As we introduce more and more detail, the situation will be represented by a larger and larger clade.

In an *event space*, we consider situations in abstraction from their representation in a particular event tree. This allows us to consider on the same footing situations that are related in a variety of ways. Two situations in an event space might be related by precedence (there is some event tree in which one precedes the other along a path down the tree). And two other situations might be related by refinement (there is some event tree where one is represented by a subset of the clade that represents the other).

Situations or instantaneous events can also be related in much more complicated ways. Consider, for example, Figure 3. This is an event tree for a shopper who sets out to buy wine, cheese, and apples. The green grocer sells all three, the cheese store sells cheese and wine, and the wine store sells only wine. The shopper has time to visit two stores at the most. The clade labelled **T** represents the instantaneous event that the shopper buys apples, while the clade consisting of the solid nodes (which we may call **S**) represents the instantaneous event that he buys wine and cheese in the same store. How do we explain the relations between these two instantaneous events?

In order to answer this question, it is helpful first to think about how a single node **S*** might be related to a clade **T**. Figure 4 reveals that there are exactly five ways S* and T can be related.

1. The single node S* is inside T. In this case, we say that the situation S* *refines* the situation T, or that whenever the instantaneous event S* happens, the instantaneous event T happens as well.

2. The single node S* is below T. In this case, we say that S* *requires* T, or that S* can only happen if T has already happened.

3. The single node S* is above T. In this case, we say that S* *foretells* T, or that if S* happens, then T will certainly happen.

4. The single node S* is partially above T. Some, but not all, branches emanating from S* lead to a situation in T. We say that S* *forebears* T, or that if S* happens, then T may happen.

5. The single node S* is outside of T. No branches emanating from S* lead to a situation in T. We say that S* *diverges* T, or that if S* happens, then T will definitely not happen.

Returning now to Figure 3, we see that in general the relation between two situations T and S can be understood in terms of how many and which of the relations in Figure 4 hold between T and parts of S. In the case of the shopping story, there are parts of S in relations 1, 2, and 3. There are no parts in relations 4 or 5. We can express this in terms of the five binary relations in Table 1: S **overlaps** T holds, S **may.Require** T holds, S **may.Foretell** T holds, S **may.forbear** T does not hold, and S **may.diverge** T does not hold.

Formally, an event space is a set $\mathcal{S}$ and five primitive binary relations with the names and intuitive meanings listed in Table 1. Certain axioms must be satisfied and certain constructions must be allowed. For lack of space, we refer the reader to [13, 14] for these axioms and constructions.

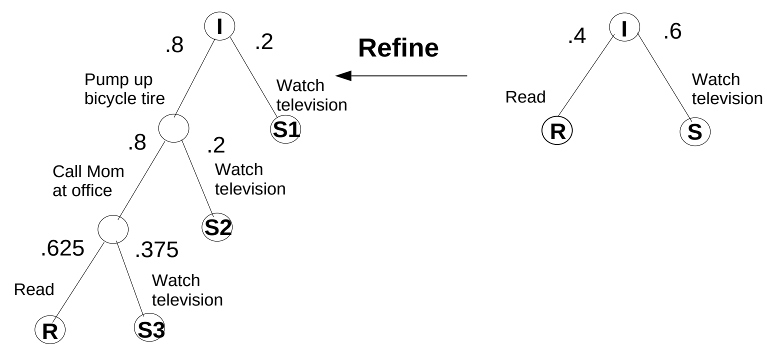

Figure 2: Refinement

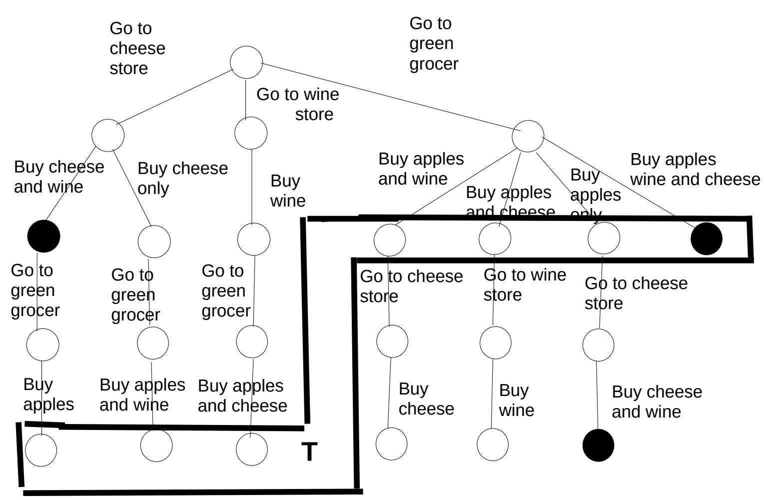

Figure 3: An event tree for a shopper

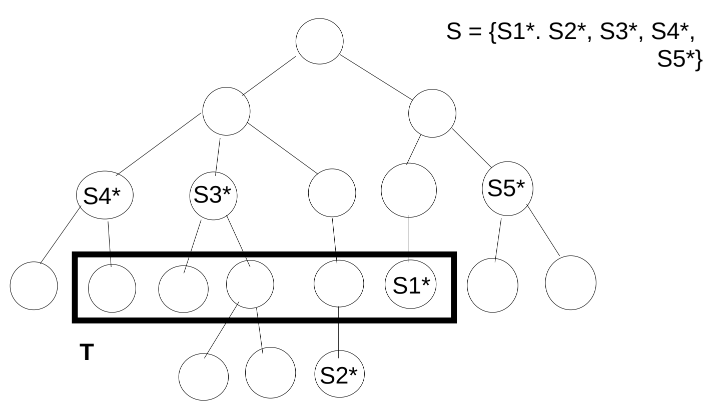

Figure 4: Five ways a single node S* can relate to a clade T

S 1 T	S **overlaps** T (S overlaps T)	S may happen in such a way that T happens at the same time.
S 2 T	S **may.Require** T (S may strictly require T)	S may happen in such a way that T has already happened.
S 3 T	S **may.Foretell** T (S may strictly foretell T)	S may happen in such a way that T must happen later.
S 4 T	S **may.forbear** T (S may strictly forbear T)	S may happen in such a way that T remains only possible.
S 5 T	S **may.diverge** T (S may diverge from T)	S may happen in such a way that T cannot happen.

Table 1: The Five Primitive Relations

The five primitive binary relations can be combined in many possible ways to yield further relations. For example, the disjunction

E1 S 1 T or S 2 T or S 3 T or S 4 T

specifies that S and T are *compatible*, meaning that events can turn out in such a way that they both happen. The negative version

E2 neither S 1 T nor S 2 T nor S 3 T nor S 4 T

states that S and T are *divergent*, meaning that if one happens the other does not.

We will use a concatenation of numerals to indicate a disjunction and a negation sign preceding the concatenated numerals to indicate the negation of the disjunction. Thus **E1** is abbreviated as S 1234 T, and **E2** is abbreviated as S ¬1234 T.

Table 2 lists a number of relations defined in terms of the primitive relations. The first five strengthen the corresponding relations in Table 1. For example, S 1 T says that S may happen in such a way that T happens at the same time, while S ¬2345 T says that this is the only way S may happen. In terms of these relations, we define S **precedes** T to mean S **allows** T and T **requires** S. When S **precedes** T, we also say that T follows S.

3 The Language

In this section, we sketch a language $\mathcal{CAL}$ for talking about actions and their effects in the context of event spaces. This language is based on the language $\mathcal{A}$ [3, 4]. Its well formed formulae represent either actions or propositions. Some interesting formulae and their intuitive meanings are given below. The symbols P, Q etc. can represent either an atomic symbol or an arbitrary formula.

1. POSS A IF $P_1, \ldots, P_N$. This is equivalent to the action precondition axioms of [11].
2. INITIALLY P.
3. P AFTER $A_1, \ldots, A_N$
4. A MAKES_TRUE P IF $P_1, \ldots, P_N$ (where $P, P_1, \ldots, P_N$ are predicates and A is an action symbol) This is equivalent to the (positive) effect axioms of [11].
5. A MAKES_FALSE P IF $P_1, \ldots, P_N$ (where $P, P_1, \ldots, P_N$ are predicates and A is an action symbol) This is equivalent to the (negative) effect axioms of [11].
6. NECESSARILY_EVENTUALLY Q IF $P_1, \ldots, P_N$ This is McCarthy's [9] fluent F. A situation that satisfies the fluents $P_1, \ldots, P_N$ will eventually be followed by one that satisfies Q.
7. POSSIBLY_EVENTUALLY Q IF $P_1, \ldots, P_N$. For every situation **S** satisfying $P_1 \ldots P_N$, there exists a **T** such that **T** may follow **S** and T satisfies Q.
8. NECESSARILY_ALWAYS Q IF $P_1 \ldots P_N$. For every situation **S** that satisfies $P_1 \ldots P_N$, for every **T** such that **T** follows **S**, T satisfies Q.
9. POSSIBLY_ALWAYS Q IF $P_1, \ldots, P_N$. For every situation S satisfying P1,...,PN and every situation T that is inevitable after S, there is a refinement T' of T that satisfies Q.

S ¬2345 T	S **refines** T (S refines T)	Whenever S happens, T happens at the same time.
S ¬1345 T	S **Requires** T (S strictly requires T)	Whenever S happens, T has already happened.
S ¬1245 T	S **Foretells** T (S strictly foretells T)	Whenever S happens, T must happen later.
S ¬1235 T	S **forbears** T (S forbears T)	Whenever S happens, T remains only possible.
S ¬1234 T	S **diverges** T (S diverges from T)	Whenever S happens, T cannot happen.
S ¬345 T	S **requires** T (S requires T)	Whenever S happens, T happens at the same time or has already happened.
S ¬245 T	S **foretells** T (S foretells T)	Whenever S happens, T happens at the same time or must happen later.
S ¬25 T	S **allows** T (S allows T)	Whenever S happens, T happens at the same time, must happen later, or remains only possible.

Table 2: The Negative Relations

10. Q ONLYIF P FIRST. In every situation that satisfies P, Q can only be true if P was true first.

11. A_1 allows A_2 if $P_1, \ldots, P_N$. For every situation **S** satisfying $P_1 \ldots P_N$, A_1 is possible and A_2 is possible in the situation resulting from A_1.

12. IF A_1 HAPPENS, THEN A_2 HAPPENS. If A_1 happens then A_2 must happen on every later path.

13. A_1 HAS_AS_PREREQUISITE A_2. If A_1 happens then A_2 happened earlier.

Further expressions are possible as well.

4 Semantics

A model **M** for our causal action language $\mathcal{CAL}$ is a pair $\langle \mathcal{S}, I \rangle$, where $\mathcal{S}$ and I are as follows:

- $\mathcal{S}$ is an event space with appropriate relations and constructions.
- I is the interpretation function, defined on the action terms and propositional symbols in $\mathcal{CAL}$. The function I takes two arguments, a situation S from $\mathcal{S}$ and an element E that is either an action term or propositional symbol of $\mathcal{CAL}$, as in $I_S(\text{E})$.
- There is a distinguished root situation Ω.
- If E is a propositional symbol, then $I_S(\text{E})$ is either TRUE, FALSE, or unspecified. Thus the logic is multivalued. Note that in a particular situation S, we allow P $\vee$ Q to be true while both P and Q are unspecified as long as either P or Q is true in every situation of the clade resulting at some level of refinement of S. This is important in properly handling nondeterministic actions. (If a situation S refines into S_1 and S_2, we say that $S = S_1 \oplus S_2$.)

  If a propositional symbol has a truth value at s_2 and s_1**refines**s_2 then that symbol must have the same truth value at s_1.

  If A is an action formula, then $I_S(\text{A})$ is a pair $\langle S, T \rangle$ where S **precedes** T. The function $I_S(\text{A})$ is partially defined. When $I_S(\text{A})$ is defined we say that A is possible at S.

Given a model **M** for the language $\mathcal{CAL}$, a situation S, the semantic value of basic expressions, indicated by $[\![\varphi]\!]^{\mathbf{M},S}$, are given below:

$$1.\quad [\![\neg\varphi]\!]^{\mathbf{M},S} \begin{array}{l} = TRUE \text{ iff } [\![\varphi]\!]^{\mathbf{M},S} = FALSE, \\ = FALSE \text{ iff } [\![\varphi]\!]^{\mathbf{M},S} = TRUE, \\ = \text{unspecified otherwise.} \end{array}$$

2.

$$
\begin{aligned}
[\![\varphi \wedge \psi]\!]^{\mathbf{M},S} &= TRUE \\
&\quad \text{iff } [\![\varphi]\!]^{\mathbf{M},S} = TRUE \text{ and } [\![\psi]\!]^{\mathbf{M},S} = TRUE \\
&= FALSE \text{ iff } [\![\varphi]\!]^{\mathbf{M},S} = FALSE \text{ or } [\![\psi]\!]^{\mathbf{M},S} = FALSE \text{ or } \\
&\quad S = S_1 \oplus S_2 \text{ and } [\![\varphi]\!]^{\mathbf{M},S_1} = FALSE \text{ and } [\![\psi]\!]^{\mathbf{M},S_2} = FALSE \\
&= \text{unspecified otherwise.}
\end{aligned}
$$

3.

$$
\begin{aligned}
[\![\varphi \vee \psi]\!]^{\mathbf{M},S} &= FALSE \\
&\quad \text{iff } [\![\varphi]\!]^{\mathbf{M},S} = TRUE \text{ and } [\![\psi]\!]^{\mathbf{M},S} = TRUE \\
&= TRUE \text{ iff } [\![\varphi]\!]^{\mathbf{M},S} = TRUE \text{ or } [\![\psi]\!]^{\mathbf{M},S} = TRUE \text{ or } \\
&\quad S = S_1 \oplus S_2 \text{ and } [\![\varphi]\!]^{\mathbf{M},S_1} = TRUE \text{ and } [\![\psi]\!]^{\mathbf{M},S_2} = TRUE \\
&= \text{unspecified otherwise.}
\end{aligned}
$$

4.

$$
\begin{aligned}
[\![\varphi \rightarrow \psi]\!]^{\mathbf{M},S} &= FALSE \\
&\quad \text{iff } [\![\varphi]\!]^{\mathbf{M},S} = TRUE \text{ and } [\![\psi]\!]^{\mathbf{M},S} = FALSE \\
&= TRUE \text{ iff } [\![\varphi]\!]^{\mathbf{M},S} = FALSE \text{ or } [\![\psi]\!]^{\mathbf{M},S} = TRUE \text{ or } \\
&\quad S = S_1 \oplus S_2 \text{ and } [\![\varphi]\!]^{\mathbf{M},S_1} = FALSE \text{ and } [\![\psi]\!]^{\mathbf{M},S_2} = TRUE \\
&= \text{unspecified otherwise.}
\end{aligned}
$$

The semantic value of the expressions of $\mathcal{CAL}$ listed in the preceding section, are defined [1] as follows:

1. $[\![$POSS A IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and $[\![P_n]\!]^{\mathbf{M},S} = TRUE$, then there is some T $\in \mathcal{S}$ such that $I_S(\text{A}) = \langle S, T \rangle$.
2. $[\![$INITIALLY P$]\!]^{\mathbf{M}} = TRUE$ if $[\![P_1]\!]^{\mathbf{M},\Omega} = TRUE$.
3. $[\![$P AFTER $\text{A}_1, \ldots, \text{A}_N]\!]^{\mathbf{M}} = TRUE$ if there is some $S_1 \in \mathcal{S}$ such $I_\Omega(\text{A}_1) = \langle \Omega, S_1 \rangle$, and ... $I_{S_{n-1}}(\text{A}_N) = \langle S_{n-1}, S_n \rangle$, and $[\![P]\!]^{\mathbf{M},S_n} = TRUE$.
4. $[\![$A MAKES_TRUE P IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that $I_S(\text{A}) = \langle S, T \rangle$ and $[\![P]\!]^{\mathbf{M},T} = TRUE$.
5. $[\![$A MAKES_FALSE P IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that $I_S(\text{A}) = \langle S, T \rangle$ and $[\![P]\!]^{\mathbf{M},T} = FALSE$.
6. $[\![$NECESSARILY_EVENTUALLY Q IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that S **foretells** T and $[\![Q]\!]^{\mathbf{M},T} = TRUE$.
7. $[\![$POSSIBLY_EVENTUALLY Q IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that S **allows** T and $[\![Q]\!]^{\mathbf{M},T} = TRUE$.
8. $[\![$NECESSARILY_ALWAYS Q IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, then for every $T \in \mathcal{S}$ such that S **precedes** T, $[\![Q]\!]^{\mathbf{M},T} = TRUE$.
9. $[\![$POSSIBLY_ALWAYS Q IF $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, then for all $T \in \mathcal{S}$ such that S **precedes** T and S **foretells** T, there is some $T' \in \mathcal{S}$ such that T **refines** T' and $[\![Q]\!]^{\mathbf{M},T'} = TRUE$.
10. $[\![$Q ONLYIF P FIRST$]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P]\!]^{\mathbf{M},S} = TRUE$, and $[\![Q]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that S **requires** T and $[\![P]\!]^{\mathbf{M},T} = TRUE$
11. $[\![\text{A}_1$ allows A_2 if $P_1, \ldots, P_N]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $[\![P_1]\!]^{\mathbf{M},S} = TRUE$ and ... and ...$[\![P_n]\!]^{\mathbf{M},S} = TRUE$, there is some $T \in \mathcal{S}$ such that $I_S(\text{A}_1) = \langle S, T \rangle$, and there is some T' such that $I_T(\text{A}_2) = \langle T, T' \rangle$.
12. $[\![$ IF A_1 HAPPENS, THEN A_2 HAPPENS $]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $I_S(\text{A}_1) = \langle S, T \rangle$, there is some $T', T'' \in \mathcal{S}$ such that $I_{T'}(\text{A}_2) = \langle T', T'' \rangle$ and T **Foretells** T'.

[1] Due to lack of space only the TRUE truth value is given.

13. $[\![\text{A}_1 \text{ HAS\_AS\_PREREQUISITE } \text{A}_2]\!]^{\mathbf{M}} = TRUE$ if for all $S \in \mathcal{S}$ such that $I_S(\text{A}_1) = \langle S, T \rangle$, there is some $S', S'' \in \mathcal{S}$ such that $I_{S''}(\text{A}_2) = \langle S'', S' \rangle$ and S **Requires** S''.

Other expressions of $\mathcal{CAL}$ can be defined in a similar fashion.

5 Conclusions and Future Work

In this paper, we have proposed an ontology that integrates action, temporal structure, and probability. The ontology is a mathematical structure called an event space. The space is inhabited by situations or instantaneous events with binary relations over them. The temporal relations within an event space include several relations that different authors have labeled "cause."

We have also proposed a propositional logic to talk about event spaces. Current work includes a variety of extensions to the work reported here, e.g., adding variables and quantifiers, developing inference rules, adding clock time (discrete or continuous), and making use of the probabilities found in the event space.

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