QUALITATIVE MARKOV NETWORKS

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I. Introduction

Students of management of uncertainty in expert systems have devoted considerable attention to propagation of belief functions and probabilities in networks (see, for example, Gordon and Shortliffe (1985), Shafer and Logan (1985), Pearl (1986)). In Shafer, Shenoy and Mellouli (1986) and Shenoy, Shafer and Mellouli (1986), a scheme for propagating belief functions in "qualitative Markov trees" is described. This scheme is a generalization of both Shafer and Logan's scheme for hierarchical trees and Pearl's scheme for Bayesian causal trees (see Shenoy and Shafer (1986)). In this paper, we concentrate on qualitative Markov trees and their properties. We start with a definition of conditional qualitative independence (q-independence) for partitions. We treat partitions as qualitative descriptions of belief functions and random variables. Using the concept of conditional q-independence, we define a qualitative Markov (q-Markov) network, analogous to a probabilistic Markov network (see, for example, Griffeath (1976), Darroch, Lauritzen and Speed (1980)). We then introduce the concept of a Kong pattern for a collection of partitions (see Kong (1986)) and note that a tree of partitions is q-Markov if and only if its edges form a Kong pattern for the partitions. For more general networks of partitions, we note that if a set of complete subsets of nodes of the network forms a Kong pattern for the partitions, then the network is q-Markov. The paper ends with some comments on Shafer, Shenoy and Mellouli's (1986) propagation scheme for belief functions in q-Markov trees.

II. Qualitative Independence for Partitions

In this paper, we will be concerned with a finite indexed collection of partitions $\{\mathcal{P}_j | j \in J\}$ of a finite nonempty set

$\Omega=\{\omega_i|i\in I\}$. Such partitions can serve as qualitative descriptions of random variables or belief functions. To a random variable $X:\Omega\rightarrow R$, we associate the partition

$$\wp_x=\{P\in 2^{\Omega}|P=X^{-1}(a) \text{ for some } a\in X(\Omega)\}$$

and to a belief function on a frame of discernment Ω, we associate the partition generated by taking intersections of the belief function's focal elements (see Shafer (1976)).

Let $\wp_1$ and $\wp_2$ be two distinct partitions. We say that $\wp_1$ is *coarser* than $\wp_2$ (or equivalently that $\wp_2$ is *finer* than $\wp_1$), written as $\wp_1\geq\wp_2$, if for each $P_2\in\wp_2$, there exists $P_1\in\wp_1$ such that $P_1\supseteq P_2$. We call $\wp_1$ a *coarsening* of $\wp_2$ and $\wp_2$ a *refinement* of $\wp_1$. The relation $\geq$ is a partial order and the set of all partitions is a lattice with respect to this partial order (Birkhoff (1967)). The *coarsest common refinement* of $\wp_1, \ldots, \wp_n$, or the least upper bound of $\wp_1, \ldots, \wp_n$ with respect to $\geq$, denoted by $\wedge\{\wp_j|j=1, \ldots, n\}$ or by $\wp_1\wedge\ldots\wedge\wp_n$, is the partition

$$\{P_1\cap\ldots\cap P_n|P_j\in\wp_j \text{ for } j=1, \ldots, n, \text{ and } P_1\cap\ldots\cap P_n\neq\emptyset\}.$$

We say that $\wp_1, \ldots, \wp_n$ are *qualitatively independent* (q-independent), written as $[\wp_1, \ldots, \wp_n]\dashv$, if whenever we select $P_j\in\wp_j$ for $j=1, \ldots, n$, we find that $P_1\cap\ldots\cap P_n\neq\emptyset$. Furthermore, we say that $\wp_1, \ldots, \wp_n$ are *conditionally q-independent* given $\wp$, written as $[\wp_1, \ldots, \wp_n]\dashv\wp$, if whenever we select $P\in\wp$ and $P_j\in\wp_j$ such that $P\cap P_j\neq\emptyset$ for $j=1, \ldots, n$, then $P\cap P_1\cap\ldots\cap P_n\neq\emptyset$. Notice that stochastic conditional independence implies qualitative conditional independence. If $\Omega=\{\omega_i|i\in I\}$ represents a finite sample space, and $Pr:2^{\Omega}\rightarrow[0,1]$ represents a probability distribution on Ω such that $Pr(\{\omega_i\})>0$ for all $i\in I$, and X, Y, Z are random variables such that X and Y are conditionally independent given Z, then $[\wp_x, \wp_y]\dashv\wp_z$, where $\wp_x$, $\wp_y$ and $\wp_z$ are the partitions associated with X, Y, and Z, respectively.

III. Qualitative Markov Networks

We now consider networks where the nodes represents partitions and the edges represent certain conditional q-independence restrictions on the partitions. Consider a *network* (J,E), where J is a finite set of partitions thought of as the *nodes* of the network, and $E\subseteq J\times J$ is a set of unordered pairs of distinct elements of J, thought of as the *edges* of the network. We say that $i\in J$ and $j\in J$ are *adjacent* or *neighbors* if $\{i,j\}\in E$. If $J_1\subseteq J$, the *boundary* of J_1, written as ∂J_1, is the set of nodes in $J-J_1$ that are adjacent to some node in J_1. The *closure* of J_1 is $J_1\cup\partial J_1$ and is denoted by $\overline{J_1}$. A *complete subset* of nodes is a subset $J_1\subseteq J$ where all elements are mutual neighbors.

A *q-Markov network* for $\{\wp_j|j\in J\}$ is a network (J,E) such that for any three mutually disjoint subsets J_1, J_2 and J_3 of J, if J_1 and J_2 are separated by J_3 (in the sense that any path from a node in J_1 to a node in J_2 goes via some node in J_3), then

$$[\wedge\{\wp_j|j\in J_1\}, \wedge\{\wp_j|j\in J_2\}]\dashv\wedge\{\wp_j|j\in J_3\}.$$

If (J,E) is a q-Markov network for $\{\wp_j|j\in J\}$ and the network (J,E) is a tree, then we say that (J,E) is a *q-Markov tree* for $\{\wp_j|j\in J\}$.

Theorem 1 (Mellouli (1987)): (J,E) is a q-Markov network for $\{\wp_j|j\in J\}$ if and only if for all $J_1\subseteq J$,

$$[\wedge\{\wp_j|j\in J_1\}, \wedge\{\wp_j|j\in J-\overline{J_1}\}]\dashv\wedge\{\wp_j|j\in\partial J_1\}.$$

Let $\{\wp_i | i \in J\}$ be an indexed collection of partitions. Let $E \subseteq 2^J$. E is said to be a *Kong pattern* for $\{\wp_i | i \in J\}$ if whenever we select an element P_i from $\wp_i$ for each $i \in J$ such that $\cap\{P_i | i \in I\} \neq \emptyset$ for all $I \in E$, then $\cap\{P_i | i \in J\} \neq \emptyset$. Notice that if the partitions in $\{\wp_i | i \in J\}$ are q-independent, then every $E \subseteq 2^J$ (including the empty set) is a Kong pattern for $\{\wp_i | i \in J\}$. Also, the singleton $\{J\}$ is always a Kong pattern for $\{\wp_i | i \in J\}$ regardless of how the $\wp_i$ are chosen. It can be shown that if E is a Kong pattern for $\{\wp_i | i \in J\}$, then (J,E) is a q-Markov network. (In fact, if a set of complete subsets of nodes in (J,E) is a Kong pattern for $\{\wp_i | i \in J\}$, then (J,E) is a q-Markov network for $\{\wp_i | i \in J\}$). The converse of this result is not valid for networks in general, i.e., neither the set of all edges nor the set of all complete subsets of nodes of a q-Markov network necessarily form a Kong pattern.

IV. Qualitative Markov Trees

We now turn our attention to the case of q-Markov trees. Two characterizations of q-Markov trees are as follows.

Theorem 2 (Shafer, Shenoy and Mellouli (1986)): Let $\{\wp_j | j \in J\}$ be a finite collection of partitions, and let (J,E) be a tree. Then (J,E) is q-Markov for $\{\wp_j | j \in J\}$ if and only if for every $j \in J$,

$$[\wedge\{\wp_i | i \in J_1\}, \ldots, \wedge\{\wp_i | i \in J_k\}] \dashv \wp_j$$

whenever $J_1, \ldots, J_k$ are subsets of J separated by $\{j\}$ in the tree (J,E).

Theorem 3 (Mellouli (1987)): Let $\{\wp_j | j \in J\}$ be a finite collection of partitions, and let (J,E) be a tree. Then

(J,E) is q-Markov for $\{\wp_i | i \in J\}$ if and only if E is a Kong pattern for $\{\wp_i | i \in J\}$.

V. Conclusion

In this paper, we focused on characterizing q-Markov trees. Elsewhere, we have described a scheme for propagating belief functions in such trees with only "local computations". The local computation aspect of the scheme results in a reduction in the computational complexity associated with Dempster's rule of combination for belief functions and also makes possible an implementation in parallel that further reduces the time required for the computation. This computational scheme is a generalization of both Shafer and Logan's scheme for hierarchical trees and Pearl's scheme for Bayesian causal trees.

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THE PRINCIPLE OF MINIMUM SPECIFICITY AS A BASIS FOR EVIDENTIAL REASONING

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Abstract : The framework of evidence theory is used to represent uncertainty pervading a set of statements which refer to subsets of a universe. Grades of credibility and plausibility attached to statements specify a class of bodies of evidence. Using newly appeared measures of specificity, a principle is stated in order to select, among these bodies of evidence, the one which suitably represents the available information in the least arbitrary way. It is shown that this principle, which is similar to the maximum entropy principle, leads to a deductive reasoning approach under uncertainty, and also provides a rule of combination which does not presuppose any independence assumption. Particularly, it is more general than Dempster's.

1 - Introduction

Recently, the idea of measure of information stemming from Shannon has been enlarged in the framework of Shafer's evidence theory. Measures of imprecision, dissonance and others (see Klir [9], Dubois-Prade [7] for complementary surveys) have been attached to a body of evidence, viewed as an allocation of probability m to subsets of a given set Ω called a frame of discernment. Namely m is such that $m(A) \geq 0$, $\forall A \subseteq \Omega$ and

$$\sum_{\emptyset \neq A \subseteq \Omega} m(A) = 1 \qquad (1)$$

The pair (F, m) where $F = \{A \mid m(A) > 0\}$ is the set of focal elements, is called a body of evidence describing the possible location of a variable x ranging on Ω. m is called a basic (probability) assignment.

This way of describing uncertain information seems to be rather ge-

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